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Well-posed Stokes/Brinkman and Stokes/Darcy problems for coupled fluid-porous viscous flows[1]

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Abstract.
We present a well-posed model for the Stokes/Brinkman problem with a family of *jump embedded boundary conditions (J.E.B.C.)* on an immersed interface with weak regularity assumptions. It is issued from a general framework recently proposed for fictitious domain problems. Our model is based on algebraic transmission conditions combining the stress and velocity jumps on the interface Σ separating the fluid and porous domains. These conditions, well chosen to get the coercivity of the operator, are sufficiently general to get the usual immersed boundary conditions on Σ when fictitious domain methods are concerned: Stefan-like, Robin (Fourier), Neumann or Dirichlet... Moreover, the general framework allows to prove the global solvability of some models with physically relevant stress or velocity jump boundary conditions for the momentum transport at a fluid-porous interface. The Stokes/Brinkman problem with *Ochoa-Tapia & Whitaker (1995)* interface conditions and the Stokes/Darcy problem with *Beavers & Joseph (1967)* conditions are both proved to be well-posed by an asymptotic analysis. Up to our knowledge, only the Stokes/Darcy problem with *Saffman (1971)* approximate interface conditions was known to be well-posed.

Keywords: Transmission problems; jump embedded boundary conditions; Stokes/Brinkman problem; Stokes/Darcy problem; fluid/porous coupled flows; well-posedness analysis; asymptotic analysis; vanishing viscosity; singular perturbation.
PACS: 02.30.Jr, 02.60.Lj, 45.10.Db, 47.10.Ad, 47.56.+r – **MSC (2000):** 35J25, 35J55, 35Q35, 65J20, 76D03, 76D07 76M45, 76S05

1. COUPLED FLUID-POROUS VISCOUS FLOWS

Notations. Let the domain $\Omega \subset \mathbb{R}^d$ ($d=2$ or 3 in practice) be an open bounded and Lipschitz continuous domain. Let an interface $\Sigma \subset \mathbb{R}^{d-1}$, Lipschitz continuous, separate Ω into two disjoint connected subdomains: the fluid domain Ω_f and the porous one Ω_p such that $\Omega = \Omega_f \cup \Sigma \cup \Omega_p$; see Fig. 1. For any quantity ψ defined all over Ω, the restrictions on Ω_f and Ω_p are denoted by ψ^f and ψ^p respectively. For a function ψ in $H^1(\Omega_f \cup \Omega_p)$, let ψ^- and ψ^+ be the traces of $\psi_{|\Omega_p}$ and $\psi_{|\Omega_f}$ on each side of Σ respectively, $\overline{\psi}_{|\Sigma} = (\psi^+ + \psi^-)/2$ the arithmetic mean of traces of ψ, and $[\![\psi]\!]_\Sigma = (\psi^+ - \psi^-)$ the jump of traces of ψ on Σ oriented by $\mathbf{n}$.

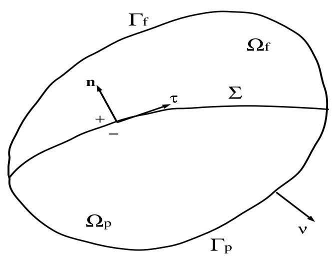

FIGURE 1. Configuration for fluid-porous flows inside the domain $\Omega = \Omega_f \cup \Sigma \cup \Omega_p$.

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There exist in the literature different models with physically relevant stress or velocity jump boundary conditions for the tangential momentum transport at the fluid-porous interface Σ, see e.g. [20, 14, 18]. When the homogeneous porous flow is to be governed by the Brinkman equation, cf. [1, 2, 8, 10, 15], the interface condition below linking the jump of shear stress with a continuous velocity was derived with volume averaging techniques by Ochoa-Tapia and Whitaker [19] instead of the usual stress and velocity continuity boundary conditions at the interface [2]:

$$\left(\mu \nabla \mathbf{v}^f \cdot \mathbf{n} - \frac{\mu}{\phi} \nabla \mathbf{v}^p \cdot \mathbf{n}\right)_{\Sigma} \cdot \tau = \frac{\mu\, \beta_{otw}}{\sqrt{K}} \mathbf{v}_{\Sigma} \cdot \tau \quad \text{and} \quad \mathbf{v}^f = \mathbf{v}^p = \mathbf{v}_{\Sigma} \quad \text{on } \Sigma, \tag{1}$$

where the dimensionless parameter β_{otw} is of order one; see [14, 12, 22] for its characterization. We show below, as a by-product of our general framework [5] recalled in the next Section that stress jump interface conditions of this type yield a well-posed fluid-porous Stokes/Brinkman problem whatever the dimensionless parameter $\beta_{otw} \geq 0$; see [6] for more details. This was not already stated up to our knowledge.

When the porous flow is governed by the Darcy equation, see e.g. [15], the well-known Beavers and Joseph interface condition [9] must be used. It links the shear stress at the interface with the jump of tangential velocity:

$$(\mu \nabla \mathbf{v}^f \cdot \mathbf{n})_{|\Sigma} \cdot \tau = \frac{\mu\, \alpha_{bj}}{\sqrt{K}} \left(\mathbf{v}^f - \mathbf{v}^p\right)_{\Sigma} \cdot \tau \quad \text{and} \quad \mathbf{v}^f \cdot \mathbf{n} = \mathbf{v}^p \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{n}_{\Sigma} \quad \text{on } \Sigma, \tag{2}$$

where the dimensionless parameter $\alpha_{bj} = \mathscr{O}(\frac{1}{\sqrt{\phi}})$ depends on the porosity ϕ and may vary between 0.1 and 4. The approximate Saffman interface condition [21], derived by homogenization techniques in [16], is also written when the porous filtration velocity can be neglected with respect to the fluid velocity at the interface: $|\mathbf{v}^p_{\Sigma} \cdot \tau| \ll |\mathbf{v}^f_{\Sigma} \cdot \tau|$, *i.e.* for a permeability value K or Darcy number $\mathrm{Da} = K/H^2$ sufficiently small. The global solvability of the Stokes/Darcy problem with the Saffman condition for $\mathbf{v}^p_{\Sigma} \cdot \tau \approx 0$ is proved with a mixed hybrid formulation in [17] whatever the dimensionless parameter $\alpha_{bj} \geq 0$, and then by many others with various formulations, see e.g. the recent review [13]. The only result of well-posedness for the full form of Beavers and Joseph condition is recently established in [11] for α_{bj}^2 sufficiently small. We show further by a singular perturbation in our general framework with a vanishing viscosity that the above Beavers and Joseph interface conditions yield a well-posed Stokes/Darcy problem whatever the parameter $\alpha_{bj} \geq 0$; see [6] for more details. Here, the main difficulty lies in how to give a sense to the tangential trace of the porous velocity on the interface with minimal regularity assumptions. This is particularly relevant for thin fluid layers as for conducting fractures in porous media flows [7, 11].

2. WELL-POSED MODELS FOR COUPLED FLUID-POROUS VISCOUS FLOWS

We first describe the general framework with jump embedded boundary conditions proposed and studied in [5] which is also useful for fictitious domain methods in a similar way as in [4]. It is derived by a generalization to vector elliptic systems of a previous model stated for scalar problems [3, 4].

2.1. Stokes/Brinkman transmission problem with jump embedded boundary conditions

Let $\sigma(\mathbf{v}, p) \equiv -p\,\mathbf{I} + 2\tilde{\mu}\,\mathbf{d}(\mathbf{v})$ denote the Newtonian stress tensor defined with the effective viscosity $\tilde{\mu}$ in the porous domain Ω_p, with $\tilde{\mu} = \mu$ in the fluid domain Ω_f and $\mathbf{d}(\mathbf{v}) \equiv \frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^t)$ being the strain rate tensor. We consider the following Stokes/Brinkman problem including *jump embedded boundary conditions (J.E.B.C.)* [5] on the interface Σ which link the trace jumps of both the stress vector $\sigma(\mathbf{v}, p) \cdot \mathbf{n}$ and the velocity vector $\mathbf{v}$ through the interface Σ:

$$-\nabla \cdot \sigma(\mathbf{v}, p) = \mathbf{f} \qquad \text{in } \Omega_f, \tag{3}$$

$$-\nabla \cdot \sigma(\mathbf{v}, p) + \mu\, \mathbf{K}^{-1} \mathbf{v} = \mathbf{f} \qquad \text{in } \Omega_p, \tag{4}$$

$$\nabla \cdot \mathbf{v} = 0 \qquad \text{in } \Omega_f \cup \Omega_p, \tag{5}$$

$$\mathbf{v} = 0 \qquad \text{on } \Gamma_f \cup \Gamma_p, \tag{6}$$

$$[\![\sigma(\mathbf{v}, p) \cdot \mathbf{n}]\!]_{\Sigma} = \mathbf{M}\, \overline{\mathbf{v}}_{|\Sigma} \qquad \text{on } \Sigma, \tag{7}$$

$$\overline{\sigma(\mathbf{v}, p) \cdot \mathbf{n}}_{|\Sigma} = \mathbf{S}\, [\![\mathbf{v}]\!]_{\Sigma} \qquad \text{on } \Sigma. \tag{8}$$

Here, the viscosity coefficient μ and effective viscosity $\tilde{\mu}$ in the porous medium are bounded positive functions such that $\mu_0 = \min(\mu, \tilde{\mu}) > 0$, the symmetric permeability tensor $\mathbf{K} \equiv (K_{ij})_{1\leq i,j\leq d}$ is uniformly positive definite, and the *transfer matrices* $\mathbf{S}$, $\mathbf{M}$ on Σ are measurable, bounded and uniformly semi-positive matrices. With usual notations for Sobolev spaces, we now define the Hilbert spaces:

$$H^1_{0\Gamma_f}(\Omega_f)^d \equiv \left\{ \mathbf{w} \in H^1(\Omega_f)^d;\ \mathbf{w}_{|\Gamma_f} = 0 \text{ on } \Gamma_f \right\}, \quad H^1_{0\Gamma_p}(\Omega_p)^d \equiv \left\{ \mathbf{w} \in H^1(\Omega_p)^d;\ \mathbf{w}_{|\Gamma_p} = 0 \text{ on } \Gamma_p \right\},$$
$$\mathbf{W} \equiv \left\{ \mathbf{w} \in L^2(\Omega)^d,\ \mathbf{w}_{|\Omega_f} \in H^1_{0\Gamma_f}(\Omega_f)^d \text{ and } \mathbf{w}_{|\Omega_p} \in H^1_{0\Gamma_p}(\Omega_p)^d;\ \nabla\cdot\mathbf{w} = 0 \text{ in } \Omega_f \cup \Omega_p \right\}$$

equipped with the natural inner product and associated norm in $H^1(\Omega_f \cup \Omega_p)^d$. The model with the J.E.B.C. (7-8) also allows a possible pressure jump $[\![p]\!]_\Sigma \neq 0$ in $H^{-\frac{1}{2}}(\Sigma)$ with additional regularity assumptions.

Then, as a consequence of the general framework stated in [5], the problem (3-8) satisfies in Ω the nice weak formulation below: *Find* $\mathbf{v} \in \mathbf{W}$ *such that* $\forall \mathbf{w} \in \mathbf{W}$, $a(\mathbf{v}, \mathbf{w}) = l(\mathbf{w})$ with

$$a(\mathbf{v},\mathbf{w}) = 2\int_{\Omega_f} \mu\, \mathbf{d}(\mathbf{v}):\mathbf{d}(\mathbf{w})\, dx + 2\int_{\Omega_p} \tilde{\mu}\, \mathbf{d}(\mathbf{v}):\mathbf{d}(\mathbf{w})\, dx + \int_{\Omega_p} \mu\, \mathbf{K}^{-1}\mathbf{v}\cdot\mathbf{w}\, dx + \int_\Sigma \mathbf{M}\overline{\mathbf{v}}_{|\Sigma}\cdot\overline{\mathbf{w}}_{|\Sigma}\, ds + \int_\Sigma \mathbf{S}\, [\![\mathbf{v}]\!]_\Sigma\cdot[\![\mathbf{w}]\!]_\Sigma\, ds$$
$$l(\mathbf{w}) = \int_\Omega \mathbf{f}\cdot\mathbf{w}\, dx. \tag{9}$$

Then, we proved in [5, Theorem 1.1] that the problem (3-8) with $\mathbf{f} \in L^2(\Omega)^d$ has a unique solution $(\mathbf{v}, p) \in \mathbf{W} \times L^2(\Omega)$ satisfying the weak form (9) for all $\mathbf{w} \in \mathbf{W}$.

2.2. Stokes/Brinkman problem with Ochoa-Tapia & Whitaker interface conditions

We now consider that $\tilde{\mu} = \mu/\phi$, where $\phi \in]0,1]$ is the porosity of the porous medium, and stress jump interface conditions of Ochoa-Tapia & Whitaker's type [19] like in (1), the original ones reading with $\beta_\tau = \beta_{otw}$ and $\beta_n = 0$:

$$[\![\sigma(\mathbf{v},p)\cdot\mathbf{n}]\!]_\Sigma = \mathbf{M}\mathbf{v} \quad \text{with } M_{jj} = \frac{\mu\,\beta_\tau}{\sqrt{K_\tau}},\ j = 1,\cdots,d-1,\ M_{dd} = \frac{\mu\,\beta_n}{\sqrt{K_n}} \quad \text{and} \quad [\![\mathbf{v}]\!]_\Sigma = 0 \quad \text{on } \Sigma, \tag{10}$$

where $\mathbf{M}$ is a positive diagonal matrix with $\beta_\tau, \beta_n \geq 0$ a.e. on Σ and K_τ, K_n permeability coefficients. Then, as a consequence of the general framework stated in [5], the problem (3-6,10) satisfies in Ω the weak formulation below: *Find* $\mathbf{v} \in \mathbf{V} = \{\mathbf{u} \in H^1_0(\Omega)^d;\ \nabla\cdot\mathbf{u} = 0\}$ *such that,*

$$2\int_{\Omega_f} \mu\, \mathbf{d}(\mathbf{v}):\mathbf{d}(\mathbf{w})\, dx + 2\int_{\Omega_p} \frac{\mu}{\phi}\, \mathbf{d}(\mathbf{v}):\mathbf{d}(\mathbf{w})\, dx + \int_{\Omega_p} \mu\, \mathbf{K}^{-1}\mathbf{v}\cdot\mathbf{w}\, dx + \int_\Sigma \mathbf{M}\mathbf{v}\cdot\mathbf{w}\, ds = \int_\Omega \mathbf{f}\cdot\mathbf{w}\, dx,\ \forall \mathbf{w} \in \mathbf{V}. \tag{11}$$

Theorem 2.1 (Global solvability of Stokes/Brinkman problem with OT-W)**.** *If usual ellipticity assumptions hold, the problem (3-6,10) with* $\mathbf{f} \in L^2(\Omega)^d$ *has a unique solution* $(\mathbf{v}, p) \in \mathbf{V} \times L^2(\Omega)$ *satisfying the weak form (11) for all* $\mathbf{w} \in \mathbf{V}$ *and such that* $p^f = p_0^f + C^1/2$ *and* $p^p = p_0^p - C^1/2$ *with* $p_0 \in L^2_0(\Omega)$ *and the constant* C^1 *defined by:*

$$C^1 = \frac{1}{|\Sigma|}\, \langle [\![\sigma(\mathbf{v}, p_0)\cdot\mathbf{n}]\!]_\Sigma - \mathbf{M}\mathbf{v}\,,\, \mathbf{n} \rangle_{-\frac{1}{2},\Sigma}\,.$$

We can also interpret this solution as the limit solution of the problem (3-8) with penalized velocity jumps on Σ when the penalty parameter $\varepsilon > 0$ tends to zero; see [6] for the details.

2.3. Stokes/Darcy problem with Beavers & Joseph interface conditions

We consider the problem (3-8) with the Dirichlet boundary condition (6) on Γ_p replaced by the stress boundary condition of Neumann where ν is the outward unit normal vector on Γ_p and $\mathbf{q} \in H^{-\frac{1}{2}}(\Gamma_p)^d$ given, e.g. $\mathbf{q} = -p_e\,\nu$:

$$\mathbf{v} = 0 \quad \text{on } \Gamma_f \quad \text{and} \quad \sigma(\mathbf{v}^p, p^p)\cdot\nu = -p^p\,\nu + \tilde{\mu}\nabla\mathbf{v}^p\cdot\nu = \mathbf{q} \quad \text{on } \Gamma_p. \tag{12}$$

Let us define the Hilbert space $\mathbf{W}_N$ equipped with the natural inner product and norm in $H^1(\Omega_f \cup \Omega_p)^d$:

$$\mathbf{W}_N \equiv \{\mathbf{w} \in L^2(\Omega)^d,\ \mathbf{w}_{|\Omega_f} \in H^1_{0\Gamma_f}(\Omega_f)^d \text{ and } \mathbf{w}_{|\Omega_p} \in H^1(\Omega_p)^d;\ \nabla\cdot\mathbf{w} = 0 \text{ in } \Omega_f \cup \Omega_p\}.$$

Then, as a corollary of [5, Theorem 1.1], see also [5, Theorem 2.1], the problem (3-5,7-8,12) has a unique solution $(\mathbf{v}, p) \in \mathbf{W}_N \times L^2(\Omega)$.

For any $\varepsilon > 0$, let us now consider the solution $(\mathbf{v}_\varepsilon, p_\varepsilon) \in \mathbf{W}_N \times L^2(\Omega)$ of the problem (3-5,7-8,12) with a vanishing viscosity $\tilde{\mu} = \varepsilon$ for the Brinkman problem in Ω_p. The condition (12) avoids the creation of a spurious boundary layer along Γ_p for the Darcy problem when $\varepsilon \to 0$. The J.E.B.C. (7-8) are also calibrated as follows to obtain interface conditions of Beavers & Joseph's type [9] with a jump of tangential velocity (2) allowing a possible pressure jump:

$$[\![\sigma(\mathbf{v}, p)\cdot\mathbf{n}]\!]_\Sigma = \mathbf{M}\,\overline{\mathbf{v}}_{|\Sigma} \quad \text{with} \quad M_{jj} = 0,\ j = 1, \cdots, d-1, \quad M_{dd} = \frac{\mu\,\beta_n}{\sqrt{K_n}} \qquad \text{on } \Sigma, \tag{13}$$

$$\overline{\sigma(\mathbf{v}, p)\cdot\mathbf{n}}_{|\Sigma} = \mathbf{S}\,[\![\mathbf{v}]\!]_\Sigma \quad \text{with} \quad S_{jj} = \frac{\mu\,\alpha_\tau}{\sqrt{K_\tau}},\ j = 1, \cdots, d-1, \quad S_{dd} = \frac{1}{\varepsilon} \qquad \text{on } \Sigma, \tag{14}$$

where $\mathbf{M}$, $\mathbf{S}$ are positive diagonal matrices with $\alpha_\tau = \alpha_{bj}$, $\beta_n \geq 0$ a.e. on Σ and K_τ, K_n permeability coefficients.

Let us define the Hilbert spaces

$$\mathbf{W}_{S/D} \equiv \left\{\mathbf{w} \in L^2(\Omega)^d,\ \mathbf{w}_{|\Omega_f} \in H^1_{0\Gamma_f}(\Omega_f)^d,\ \mathbf{w}_{|\Omega_p} \in L^2(\Omega_p)^d;\ \nabla\cdot\mathbf{w} = 0 \text{ in } \Omega_f \cup \Omega_p\right\}$$

equipped with the natural inner product and norm in $H^1(\Omega_f)^d \times L^2(\Omega_p)^d$ and

$$\mathbf{W}_{S\text{–}D} \equiv \left\{\mathbf{w} \in \mathbf{W}_{S/D};\ \nabla\cdot\mathbf{w} \in L^2(\Omega),\ [\![\mathbf{w}]\!]_\Sigma \in L^2(\Sigma)^d,\ [\![\mathbf{w}\cdot\mathbf{n}]\!]_\Sigma = 0\right\}$$

equipped with the norm defined by: $\|\mathbf{w}\|^2_{\mathbf{W}_{S\text{–}D}} = \|\mathbf{w}\|^2_{1,\Omega_f} + \|\mathbf{w}\|^2_{0,\Omega_p} + \|\nabla\cdot\mathbf{w}\|^2_{0,\Omega} + \|[\![\mathbf{w}]\!]_\Sigma\|^2_{0,\Sigma}$.

We prove in [6] the following convergence result which also ensures the well-posedness of the Stokes/Darcy problem with Beavers & Joseph's type interface conditions (2,13) whatever the coefficients $\alpha_\tau, \beta_n \geq 0$ a.e. on Σ.

Theorem 2.2 (Convergence to Stokes/Darcy problem with B-J)**.** *With the data* $\mathbf{f} \in L^2(\Omega)^d$ *and* $\mathbf{q} = 0$*, the solution* $(\mathbf{v}_\varepsilon, p_\varepsilon)$ *in* $\mathbf{W}_N \times L^2(\Omega)$ *for any* $\varepsilon > 0$ *of the problem (3-5,12,13,14) with a vanishing viscosity* $\tilde{\mu} = \varepsilon$ *weakly converges to the solution* $(\mathbf{v}, p)$ *in* $\mathbf{W}_{S/D} \times L^2(\Omega)$ *of the Stokes/Darcy problem with the interface conditions (2,13) on* Σ *when* $\varepsilon \to 0$*. Indeed, in the porous domain* Ω_p*,* $\mathbf{v}^p$ *and* p^p *satisfy the Darcy equation,* i.e. *Eq. (4) with* $\tilde{\mu} = 0$*, and* p^p *belongs to* $H^1(\Omega_p)$ *such that* $p^p = 0$ *on* Γ_p*.*
With additional regularity assumptions such that $\mathbf{v}^p \in H^1(\Omega_p)^d$*, then* $\mathbf{v} \in \mathbf{W}_{S\text{–}D} \cap \mathbf{W}_N$ *and we have the global error estimate with* $C > 0$ *depending on the data,* $\|\nabla\mathbf{v}\|_{0,\Omega_p}$*,* $\|\psi\|_{0,\Sigma}$ *and* ψ *defined as the weak limit of* $\frac{1}{\varepsilon}[\![\mathbf{v}_\varepsilon\cdot\mathbf{n}]\!]_\Sigma$ *in* $L^2(\Sigma)$*:*

$$||\mathbf{v}_\varepsilon - \mathbf{v}||_{1,\Omega_f} + \sqrt{\varepsilon}\,||\mathbf{v}_\varepsilon - \mathbf{v}||_{1,\Omega_p} + ||\mathbf{v}_\varepsilon - \mathbf{v}||_{0,\Omega_p} + ||p_{0\varepsilon} - p_0||_{0,\Omega} + \frac{1}{\sqrt{\varepsilon}}\|[\![\mathbf{v}_\varepsilon\cdot\mathbf{n}]\!]_\Sigma\|_{0,\Sigma} \leq C\,\|\psi\|_{0,\Sigma}\,\sqrt{\varepsilon}.$$

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